A COMPLETE ANALYSIS OF THE DISCRETE MODE SPECTRUM OF OPEN MICROSTRIP TRANSMISSION LINES

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A complete analysis of the discrete mode spectrum of open microstrip transmission lines is presented. Proposing a complete set of distribution functions for the longitudinal component of the current on the stripconductor, the dispersion characteristics of the discrete modes are derived. The discrete modes have physical acceptable and interesting properties.

Introduction

The open microstrip transmission line (fig.1) has found widespread application in the development of microwave integrated circuits. To a large extent, however, this development has been empirical due to the abscence of an exact theory for this open boundary-value problem. Till now the modal analysis was allways restricted to the fundamental mode or "kwasi-T.E.M." mode[1,2].

In this paper we present a dynamical theory including the fundamental mode as well as the modes of higher order.

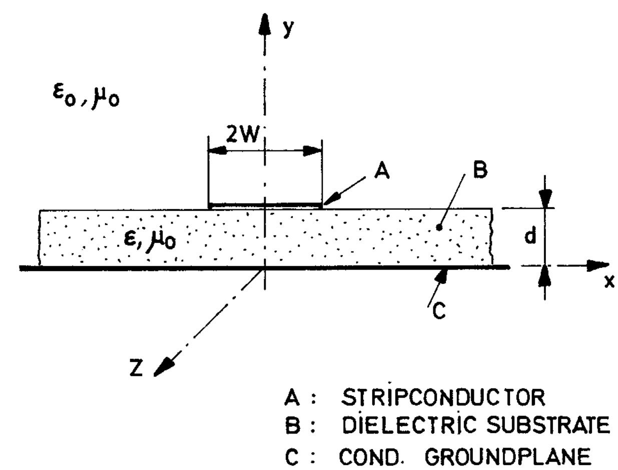

FIGURE 1 : OPEN MICROSTRIP TRANSMISSION LINE

Dynamical theory

The basical idea of this theory are the general principles formulated by G. Deschamps[3] :

1. The longitudinal wavenumber ζ has an eigenvalue spectrum consisting of two sets :
   - a finite set of discrete eigenvalues leading to a finite number of discrete modes, including the fundamental mode ;
   - a continuous sequence of eigenvalues corresponding to the radiation field.
2. The relative importance of the radiation field is only to evaluate with respect to the way of exciting the structure.

The discrete modes are no pure T.E. or T.M.-modes, but are hybride modes. Therefore we formulate the boundary-value problem applicating the Helmholtz equation to the longitudinal field components :

$$\nabla^2 E_z + k^2 . E_z = 0$$
$$\nabla^2 H_z + k^2 . H_z = 0$$

with :

$$k^2 = \omega^2 . \mu_o . \varepsilon_o \qquad \text{as : } y > d$$
$$k^2 = \omega^2 . \mu_o . \varepsilon_o . \varepsilon_r \qquad \text{as : } 0 \leqslant y \leqslant d .$$

The longitudinal field components satisfy the following boundary conditions at the plane y = 0 :

$$E_z \ (x, y = 0, z) = 0$$
$$\frac{\partial}{\partial y} H_z \ (x, y = 0, z) = 0 .$$

At the interface substrate-air the tangential field components are continuous :

$$E_{x_1} = E_{x_2}$$
$$E_{z_1} = E_{z_2}$$
$$H_{z_1} - H_{z_2} = I_x$$
$$H_{x_1} - H_{x_2} = - I_z .$$

The subscript 1 is valid for the region $y > d$ and the subscript 2 for the substrate $0 \leqslant y \leqslant d$. An equivalent, but unknown, current distribution replaces the stripconductor :

$$\bar{I}(x,y,z) = (I_z(x).\bar{i}_z + I_x(x).\bar{i}_x).\delta(y-d).\exp(-j\zeta z) \qquad \text{for : } |x| \leqslant w$$
$$\bar{I}(x,y,z) = \bar{0} \qquad \text{for : } |x| > w .$$

After a complex Fourier transformation of the real space variable x to the complex variable ξ, we apply the foregoing conditions to the field components.

The boundary conditions at the stripconductor :

$$\left. \begin{array}{l} E_z(x,y=d, z) = 0 \\ \frac{\partial}{\partial y} H_z \ (x,y=d, z) = 0 \end{array} \right\} \text{ for } |x| \leqslant w$$

only applicable in the real space domain, result in a set of two coupled integral equations .

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For each eigenvalue of the longitudinal wavenumber ζ, there is a corresponding pair of current distribution functions I_z (x) and I_x (x). These functions satisfy the couple of integral equations, while the value of ζ is given by the integral eigenvalue equation :

$$\left[\int_{\xi=-\infty}^{+\infty} K_{1,x}.I_x(\xi).\ \exp(-j\xi x).d\xi.\ \int_{\zeta=-\infty}^{+\infty} K_{2,z}.I_z(\zeta).\ \exp(-j\zeta x).d\zeta\right] -$$

$$\left[\int_{\xi=-\infty}^{+\infty} K_{2,x}.I_x(\xi).\ \exp(-j\xi x).d\xi.\ \int_{\xi=-\infty}^{+\infty} K_{1,z}.I_z(\xi).\ \exp(-j\xi x).d\xi\right] = 0.$$

The functions I_z (ξ) and I_x (ξ) are the Fourier transforms of the unknown current components I_z (x) and I_x (x).

A complete solution should give an infinite set of possible discrete surface-wave modes, of which only a finite number occur in the field representation for a given frequency.

We introduce a classification of these discrete modes in even and odd modes in correspondence with the even or odd character of $I_z(x)$. The modes are indicated with the symbol EH_n. The subscript n is the order of the mode and will be equal to the number of zeros of $I_z(x)$.

Current distribution functions

Untill now all investigators[1,2] suppose a current distribution function or use a numerical method, based upon Galerkin's procedure in the spectral domain[4], to solve this eigenvalue problem. These studies have been limited to the fundamental or EH_o-mode.

In this contribution we determine the propagation characteristics of the fundamental mode as well as the higher order modes, proposing a complete set of current distribution functions for the longitudinal component $I_z(x)$ (fig. 2 & 3). The derivation of this set of functions has been based on the Maxwell distribution function for the charge on an isolated conducting strip :

$$\sigma\ (x) = \frac{\sigma_o}{\pi}\ .\ f\ (x)$$

with

$$f\ (x) = (1 - (\tfrac{x}{w})^2)^{-1/2} \qquad \text{for : } |x| \leqslant w$$

$$= 0 \qquad \text{for : } |x| > w.$$

The current distribution function :

$$I_{z_o}\ (x) = I_{z_o}\ .\ f\ (x)$$

proposed by Denlinger[1] for the fundamental mode, gives results very good in agreement with experiments. The Fourier transform of f (x) is a zero order Bessel function of the first kind.

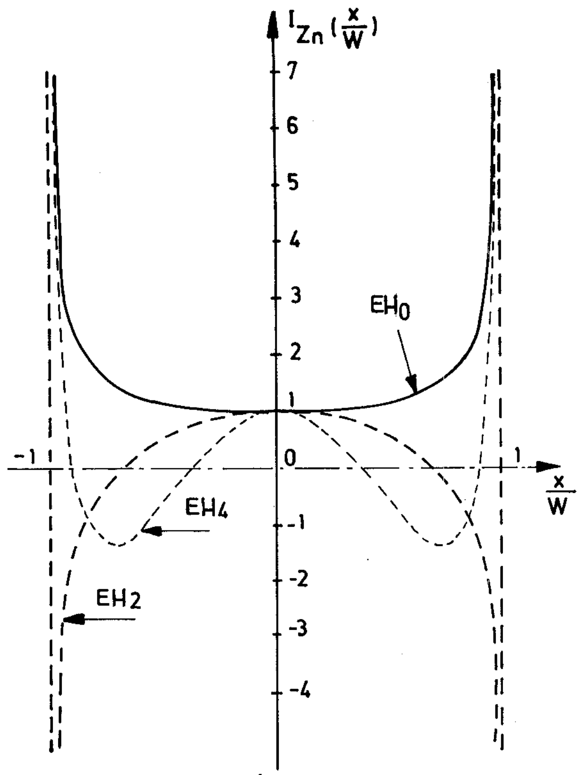

FIGURE 2 : LONGITUDINAL CURRENT DISTRIBUTION FUNCTION FOR THE EH_0, EH_2, EH_4-MODE

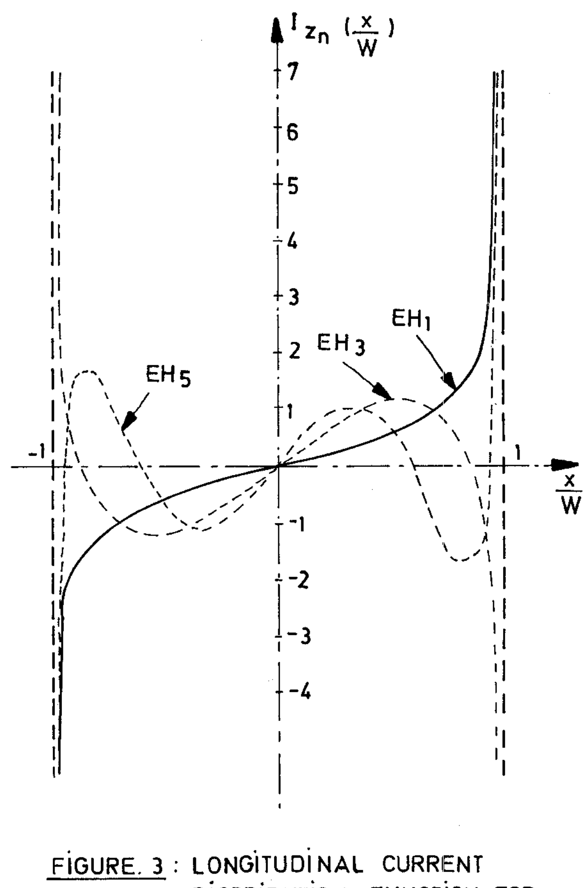

FIGURE. 3 : LONGITUDINAL CURRENT DISTRIBUTION FUNCTION FOR THE EH_1, EH_3, EH_5-MODE.

We propose a complete set, consisting of the inverse Fourier transform of the higher order Bessel functions (fig.2 & 3) as current distribution functions for the discrete modes of higher order :

$$I_{z_1}(x) = I_{z_1} \cdot \frac{x}{w} \cdot f(x)$$

$$I_{z_2}(x) = I_{z_2} \cdot \left[1 - 2\left(\frac{x}{w}\right)^2\right] \cdot f(x)$$

$$I_{z_3}(x) = I_{z_3} \cdot \left[3\left(\frac{x}{w}\right) - 4\left(\frac{x}{w}\right)^3\right] \cdot f(x)$$

$$I_{z_4}(x) = I_{z_4} \cdot \left[1 - 8\left(\frac{x}{w}\right)^2 + 8\left(\frac{x}{w}\right)^4\right] \cdot f(x)$$

$$I_{z_5}(x) = I_{z_5} \cdot \left[5\left(\frac{x}{w}\right) - 20\left(\frac{x}{w}\right)^3 + 16\left(\frac{x}{w}\right)^5\right] \cdot f(x)$$

The transversal current component $I_x(x)$ is assumed to be identically zero, since its magnitude is one order smaller than the magnitude of the longitudinal one[1].

Discrete Mode Characteristics

For the fundamental mode, the numerical results are in very good agreement with the experiments of Denlinger[1]. The phase velocity as a function of the frequency is accurate to within 0.2 % (fig.4).

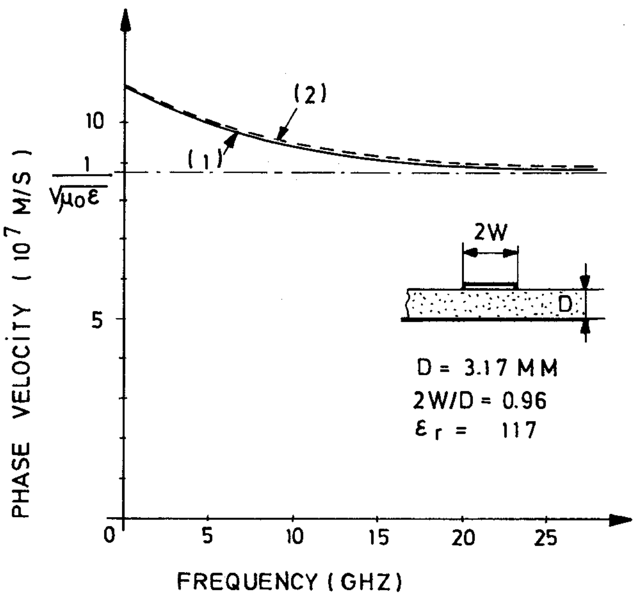

FIGURE 4 : DISPERSION CHARACTIRISTIC OF EH0 MODE
(1) THEORETICAL RESULTS
(2) EXPERIMENTS [1]

The proposed current distribution functions of higher order give convergent eigenvalues for the longitudinal wavenumber ζ. This eigenvalue has been calculated in function of the frequency. Fig. 5 shows the corresponding value of the effective relative permittivity, determined by the relation :

$$\varepsilon_{r_{eff}} = \frac{\zeta^2}{\omega^2 \cdot \mu_0 \cdot \varepsilon_0}$$

for the first four modes.

Analysing the dispersion characteristics of the discrete mode spectrum, a number of interesting properties can be deduced. For the fundamental mode or EH_0-mode, we have a convergent eigenvalue for the longitudinal wavenumber ζ starting from frequency zero. For the higher order modes, however, a lowest cut-off frequency has been found. The value of this cut-off frequency increases with the order of the mode. Below its cut-off frequency the discrete mode in question does not longer exist, while the energy excited in this mode is not longer guided along the microstripline but radiated[5]. Above its cut-off frequency, a higher order mode propagates with a phase velocity greater than the phase velocity of each mode of lower order at the same frequency. For increasing frequencies the phase velocity has a lower limit, with a value equal to the phase velocity in the dielectric medium of the substrate.

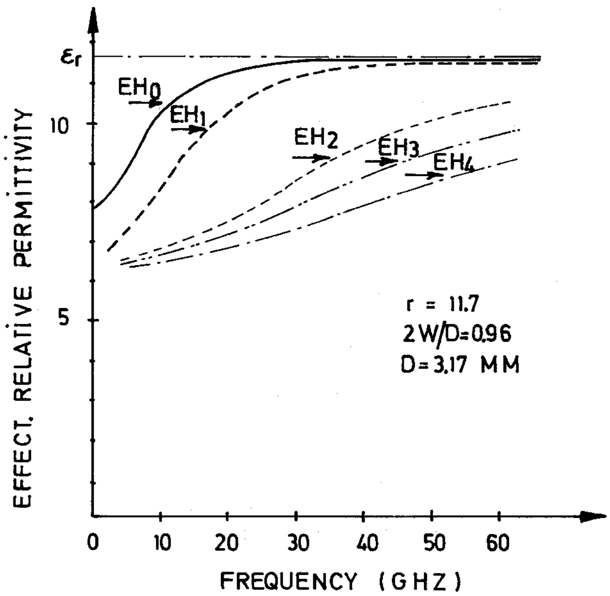

FIGURE 5 : DISPERSION CHARACTERISTIC OF EH0-EH1-EH2-EH3-EH4-MODE

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